JOURNAL OF FINANCIAL INTERMEDIATION **6,** 269–306 (1997)
ARTICLE NO. JF970223

Liars Never Prosper? How Management Misrepresentation Reduces Monitoring Costs

John C. Persons*

Fisher College of Business, Ohio State University, 1775 College Road, Columbus, Ohio 43210

Received July 16, 1996

When monitoring is not contractible—so investors monitor only when, at that time, they expect to benefit from doing so—efficient contracts sometimes induce managers to make *false* reports to investors. Because of monitoring discretion, management misrepresentation can produce Pareto improvements by reducing monitoring costs. When costs of renegotiation are small, optimal contracts necessarily induce misrepresentation. Discretionary monitoring also generates an equilibrium role for multiple-security capital structures. When an optimal contract has two investors, securityholder conflict arises endogenously as a means of reducing monitoring costs. It is efficient to write the contract so that one investor's decision to monitor hurts the other investor. *Journal of Economic Literature* Classification Number: G32.

Instances of corporate managers misrepresenting their firm's condition to investors seem to be an unpleasant fact of economic life. The Bre-X Minerals fraud is a spectacular recent example,[1] and less sensational episodes abound. While some cases may be innocent mistakes rather than deliberate deception, there can be little doubt that managers intentionally mislead investors in many of these incidents.

Why do managers sometimes lie to investors? One's immediate reaction is probably that they anticipate personal gain, notwithstanding the folk wisdom that "liars never prosper." If we then ask why managers are not

* I am grateful for helpful comments from A. Arya, B. Betker, A. Boot (the editor), S. Buser, T. George, R. Stulz, R. Young, L. Zingales, seminar participants at Duke and Ohio State, and especially two anonymous referees. Financial support was provided by the Charles A. Dice Center for Research in Financial Economics.

[1] For example, see M. Heinzl, "Bre-X, Confirming Worst Fears, Says Busang Contains Virtually No Gold," *Wall Street Journal*, May 5, 1997, C19.

1042-9573/97 $25.00

given incentive packages that induce them to tell the truth, economists will likely respond that providing such incentives would be too costly. However, the standard approach to financial contracting under asymmetric information is to invoke the Revelation Principle [Myerson (1979), Harris and Townsend (1981), Townsend (1988)] and search for the best contract that induces truthful reporting by management. The Revelation Principle shows that, under certain conditions, a truth-telling contract is *optimal*—no other contract can do better, so it is not "too costly" to provide incentives to tell the truth.

In contrast, this paper shows that false reporting by management produces Pareto improvements under plausible conditions. The basic setting is one of costly verification, as in Townsend (1979), where a manager observes the firm's cash flow but investors must incur some cost to learn the outcome.[2] The key assumptions are: (1) Managers report to investors rather than reporting confidentially to some sort of mediator or central computer. (2) Monitoring is not contractible, so investors monitor management when it "pays" to do so—they investigate a manager's profit report only when the expected payoff from investigating covers the investigation cost.

With this discretionary monitoring, setting the contractual terms in a way that induces truthful reporting is optimal for some parameters, but for others it is more efficient to induce false reporting—management misrepresentation can promote social welfare. Misrepresentation is beneficial because sometimes it is the optimal way to provide investors with appropriate incentives to monitor the manager's realized cash flow. Such incentives are unnecessary when monitoring is contractible, so false reports are not helpful in that setting. The contractual transfers can be set so that the investor receives a large transfer when she uncovers a false report, and this large potential transfer motivates her to investigate. Providing monitoring incentives in this way by keeping the investor "in the dark" about the true state of the world is sometimes more efficient than inducing truth telling by the manager and simply giving the investor a larger transfer when she verifies a truthful report. If managers sent confidential reports to a mediator, it would be optimal to induce managers to report truthfully; the mediator could perform the "scrambling" of information that misrepresentation accomplishes here, where there is direct communication between the manager and investors.

[2] There is an extensive literature of costly verification models that build on the foundations laid by Townsend (1979), including Evans (1980), Gale and Hellwig (1985), Reinganum and Wilde (1986), Graetz *et al.* (1986), Baiman *et al.* (1987), Border and Sobel (1987), Townsend (1988), Mookherjee and Png (1989), Melumad and Mookherjee (1989), Lacker (1989), Beck and Jung (1989), Chang (1990), Krasa and Villamil (1992), Boyd and Smith (1992, 1994, 1996) and Winton (1992). See the surveys in Harris and Raviv (1991) and Allen and Winton (1992).

One should not conclude that the *possibility* of false reporting improves social welfare. If false reporting were impossible, one could contract directly on the realized state, and everyone would be better off. The point is that, given the possibility of false reporting, it is sometimes socially beneficial to write contracts that induce false reporting in equilibrium.

Discretionary monitoring also generates a role for multiple securities in the firm's capital structure. When there are some fixed costs of contracting, the optimal financial structure in the model has a single security when monitoring is contractible—the only incentive problem is to persuade the manager to report appropriately, and this depends only on the total transfer to investors, so using multiple securities is inefficient. But with discretionary monitoring, using two investors gives added flexibility in providing monitoring incentives. Each investor's monitoring incentives depend on her individual transfers and the manager's reporting incentives depend on the total transfer. The transfers can be designed to give one investor the incentive to monitor, and the total transfers can be set by adjusting the other investor's transfers (as long as she is not given an incentive to monitor also). Sometimes a two-investor contract is optimal despite the higher fixed contracting costs, and sometimes a single-investor contract is optimal.[3] When a two-investor contract is used, *securityholder conflict* arises endogenously as an efficient contracting arrangement. If one investor monitors, the costs of monitoring are borne by the other investor—when Investor 1 monitors, the expected payoff of Investor 2 (rather than the manager) is reduced.

U.S. bankruptcy law provides an institutional illustration of the model's implications. Creditors' committees are allowed to recover their expenses from the firm (Betker 1995). The increased transfer provides the investors with incentives to monitor—otherwise, senior creditors would likely shut down the firm even if doing so is inefficient. Notice also that it is the other investors who bear this cost (not the manager); such conflict among securityholders emerges as an efficient arrangement in the model.

I also examine whether the optimal contracts described above can survive when the parties are able to renegotiate the contractual transfers after the manager makes his report. This change in the model reinforces the implication that misrepresentation can be beneficial. Without renegotiation, a truth-telling contract is optimal for some parameters and a misrepresentation contract is optimal for others. When renegotiation is possible and costs of renegotiation are small, a truth-telling contract does not survive renegotiation—every renegotiation-proof contract involves false reporting.

[3] Previous work that has motivated multiple-security capital structures in other ways includes Allen and Gale (1989), Harris and Raviv (1989), Winton (1992), Boot and Thakor (1993), Berglöf and von Thadden (1994), Dewatripont and Tirole (1994), Hart and Moore (1995), and Bolton and Scharfstein (1996).

As costs of renegotiation go to zero, misrepresentation becomes rampant. Virtually all reports that elicit investor monitoring are suspect, in the sense that there is a positive probability that the report is false. Thus, not only is misrepresentation (sometimes) an efficient way to provide monitoring incentives, it is also needed to prevent renegotiation, at least when costs of renegotiation are small.

The paper proceeds as follows. Section 1 lays out the model and the optimization problem to be solved. Section 2 analyzes the two-state case, which permits an explicit solution to the contracting problem and illustrates the basic points. Section 3 examines the general case of an arbitrary (but finite) number of states. Section 4 adds the possibility of renegotiation to the model. Section 5 discusses the related literature, and Section 6 contains concluding remarks. Proofs of the propositions are in the appendix.

1. THE MODEL

A risk-neutral entrepreneur has a project available that requires investment I and produces one future cash flow x. The stochastic future cash flow takes values $x_1, \ldots, x_S$, with state probabilities $\pi_1, \ldots, \pi_S$, so the project generates expected profits of $\sum_i \pi_i x_i - I$. The states are ordered $x_1 < \cdots < x_S$, and each π_i is positive. The entrepreneur has no cash to fund the project, but each investor, also risk neutral, has sufficient wealth to do so. The purpose of the contract is to get the manager to pay out enough of the project cash flows to investors that they are willing to provide the initial financing. An optimal contract accomplishes this purpose efficiently by minimizing the deadweight contracting cost.

An investor who participates in financing the firm incurs a fixed cost $f > 0$, which one can think of as the cost of evaluating the firm (i.e., determining the distribution of cash flows). Alternatively, nothing changes in the analysis if f is a cost incurred by the firm when issuing securities. There is no time preference, and there is competition among investors; therefore, any investor is willing to invest an amount equal to her expected future payment, net of up-front costs and any future monitoring costs. Let N denote the number of investors, let R^n denote the expected net revenue received by investor n, and let $R = \sum_{n=1}^{N} R^n$ be the sum across investors. The entrepreneur invests I in the project and retains any excess, but an efficient contract will raise exactly $R = I$. He also receives any cash that remains after investors are paid at the end of the project.

If the project proceeds, the entrepreneur manages it. I will therefore use the terms entrepreneur and manager interchangeably. (Perhaps the entrepreneur's project-specific human capital is the reason he has access to the project.) Because of his intimate involvement with the project, the manager observes the cash flow x. Investors, on the other hand, must incur the monitoring cost $c > 0$ if they wish to learn x. Even then, the monitoring may not succeed. If an investor chooses to monitor, she learns x (and obtains verifiable proof of x) with probability q, $0 < q < 1$, or learns nothing with probability $1 - q$.

The investor's monitoring activity is not contractible—she can always choose to shirk and avoid cost c and cannot be directly penalized for such shirking. There are several ways to motivate this assumption. One motivation is that the effort involved in monitoring is simply unobservable; one cannot tell how hard the investor is working at monitoring, so one cannot tell whether the failure to produce evidence of x is due to chance or due to shirking. Even if the monitoring activity is observable, it may not be verifiable in court, so contracts contingent on monitoring would be unenforceable. Finally, it might be that monitoring is both observable and verifiable, but the actions required for effective monitoring in particular circumstances are too complex to be specified ex ante in the contract. In this situation, contracts again cannot be made contingent on monitoring.[4]

The following restrictions on the required investment allow us to concentrate on the interesting cases. The lower bound in (1) ensures that *some* monitoring is necessary—riskless debt is insufficient—and the upper bound in (1) ensures that both truth-telling contracts and contracts with misrepresentation are feasible means of financing the project.

$$x_1 - f < I < q \sum_i \pi_i x_i + (1 - q)x_1 - f - \max\{c, c(q^{-1} - \pi_S)\}. \qquad (1)$$

I also assume that the cost of monitoring is bounded above as

$$c < \min\left\{q\left(\sum_i \pi_i x_i - x_1\right), \frac{q(\sum_i \pi_i x_i - x_1)}{q^{-1} - \pi_S}, q x_1\right\}. \qquad (2)$$

The first two bounds guarantee that the interval in (1) is nonempty and the third bound is needed for a truth-telling contract to be feasible.[5]

[4] For more on noncontractibility, see Grossman and Hart (1986) and Hart and Moore (1988, 1990).

[5] The contracting problem will be presented momentarily, but the parameter restrictions (1) and (2) guarantee that the following are feasible solutions: (a) There is one investor; the manager reports truthfully; all management reports except $\hat{x}_S$ (the highest) are investigated; when the report is $\hat{x}_S$, the transfer to the investor is $qx_S + (1 - q)x_1$; for all other reports, the transfer is x when monitoring succeeds and $x_1 - c/q$ when monitoring fails. (b) There is one investor; the manager always reports $\hat{x}_1$; all management reports are investigated; the transfer to the investor is x when monitoring succeeds and x_1 when monitoring fails.

After observing the project cash flow x, the manager reports the cash flow to investors. Since there are S possible cash flows $(x_1, \ldots, x_S)$, there are also S allowable reports $(\hat{x}_1, \ldots, \hat{x}_S)$. This permits an unambiguous interpretation of truthful reporting and allows easy comparisons with previous work. It also corresponds with SEC and FASB rules in that firms are required to report a single number for net income, e.g., not a collection of possible values—the set of reports equals the set of outcomes. The probability that the manager reports $\hat{x}_j$ when the true state is x_i is denoted σ_{ij}.

After observing the manager's report, each investor decides whether to monitor at cost c. If no one investigates the manager's report or if monitoring fails, the transfers to investors are a function of the manager's report, with $t_j^n = t^n(\hat{x}_j)$ representing the payment to investor n. If some investor monitors successfully, the transfer can be a function of both the report and the actual state. Suppose the manager reports cash flow $\hat{x}_j$ and some investor monitors and learns that the true cash flow is x_i. Then the payment to investor n is denoted $v_{ij}^n = v^n(x_i, \hat{x}_j)$. Since the manager has no wealth, the transfer can never be larger than x_i. I also assume that investors have limited liability—they cannot be required to make future investments, so the transfers are nonnegative.[6]

Let p_j^n denote the probability that investor n monitors when the manager reports $\hat{x}_j$, and let p_j denote the probability that some investor monitors. Because monitoring is not contractible, this monitoring probability must be consistent with investor incentives: if some investor has a strict incentive to monitor, then $p_j = 1$, and if every investor has a strict incentive to avoid monitoring, then $p_j = 0$. I assume that if more than one investor monitors, they all succeed (with probability q) or all fail (with probability $1 - q$). That is, monitoring success is perfectly correlated across investors. This assumption simplifies the analysis and makes it less likely that using multiple investors will be beneficial.[7] With probability $p_j q$, monitoring succeeds and the tranfers are v_{ij}^n. With probability $1 - p_j q$, the transfers are t_j^n because no evidence emerges to confirm or contradict the manager's report—either the investor does not monitor (probability $1 - p_j$) or monitoring fails (probability $p_j(1 - q)$).

[6] Note that if *any* investor monitors successfully, all investors receive the transfer v_{ij}^n rather than t_j^n. This could be the result of a judicial proceeding that enforces the transfer schedule v. Another possibility is that transfers from the firm to investors are publicly observed, so when the monitoring investor receives v_{ij}^n rather than t_j^n, everyone knows an investigation has occurred. Thus, the motivation for multiple securities in Winton (1992), to avoid duplicating verification costs, is absent here.

[7] Suppose instead that monitoring success is independent across investors. Then if two investors monitor, the probability that at least one of them succeeds, triggering transfers v_{ij}^n rather than t_j^n, increases to $1 - (1 - q)^2 > q$. Thus, using multiple investors could help by making monitoring more precise.

The manager chooses his reporting strategy σ_{ij} to maximize his expected payoff, given the terms of the contract $\{t_j^n, v_{ij}^n\}$ and investors' monitoring strategies $\{p_j^n\}$. He does so by minimizing the expected transfer to investors, so σ_{ij} can be positive only if report $\hat{x}_j$ minimizes the expected transfer $p_j q \sum_n v_{ij}^n + (1 - p_j q) \sum_n t_j^n$ when the true cash flow is x_i. This minimization is over all feasible reports $\hat{x}_k$ with $\sum_n t_k^n \leq x_i$.

The expected net revenue to investor n is the difference between the expected transfer from the firm and the expected contracting costs, both fixed costs and monitoring costs,

$$R^n = \sum_i \pi_i \sum_j \sigma_{ij}[p_j q v_{ij}^n + (1 - p_j q) t_j^n - p_j^n c] - f.$$

Since R^n is the amount investor n is willing to pay for her security, and the entrepreneur must raise at least I to fund the project, a feasible contract must have

$$R = \sum_n R^n = \sum_i \pi_i \sum_j \sigma_{ij} \left[p_j q \sum_n v_{ij}^n + (1 - p_j q) \sum_n t_j^n - c \sum_n p_j^n \right] - Nf \geq I.$$

One final bit of notation is needed. Let γ_{ij} denote the probability that the true state is x_i, given that report $\hat{x}_j$ has been made, i.e.,

$$\gamma_{ij} = \begin{cases} 0 & \text{if } \sigma_{ij} = 0 \\ \pi_i \sigma_{ij} / \sum_k \pi_k \sigma_{kj} & \text{if } \sigma_{ij} > 0. \end{cases} \tag{3}$$

If there is an investor whose expected payoff from investigating report $\hat{x}_j$ increases by more than the cost of monitoring, $q \sum_i \gamma_{ij}(v_{ij}^n - t_j^n) > c$, then p_j must be 1; otherwise, this investor would increase p_j^n. If $q \sum_i \gamma_{ij}(v_{ij}^n - t_j^n) < c$ for every investor, then p_j must be 0, since each investor will set $p_j^n = 0$. If neither condition applies, any p_j between 0 and 1 can be supported. If the contract induces truthful reporting, we have $\sigma_{ii} = \gamma_{ii} = 1$ for all i, and the conditions simplify to comparing $q(v_{ii}^n - t_i^n)$ with c.

The entrepreneur designs the contract to maximize his wealth. The program, labeled Problem 1, is to choose nonnegative numbers N, $\{t_j^n\}$, $\{v_{ij}^n\}$, $\{p_j^n\}$, $\{p_j\}$, and $\{\sigma_{ij}\}$ to solve

Problem 1.

$$\max R - I + \sum_i \pi_i \sum_j \sigma_{ij} \left[x_i - p_j q \sum_n v_{ij}^n - (1 - p_j q) \sum_n t_j^n \right]$$

subject to

$$R = \sum_i \pi_i \sum_j \sigma_{ij} \left[p_j q \sum_n v_{ij}^n + (1 - p_j q) \sum_n t_j^n - p_j c \right] - Nf \geq I. \tag{4}$$

$$\text{For each } i, \sum_j \sigma_{ij} = 1. \tag{5}$$

For each (i, j), $\sigma_{ij} > 0$ only if for all k with $\sum_n t_k^n \leq x_i$,

$$p_j q \sum_n v_{ij}^n + (1 - p_j q) \sum_n t_j^n \leq p_k q \sum_n v_{ik}^n + (1 - p_k q) \sum_n t_k^n. \tag{6}$$

$$\text{For each } j, \sum_n p_j^n = p_j \leq 1. \tag{7}$$

$$\text{For each } j, p_j = 1 \text{ if there is some } n \text{ with } q \sum_i \gamma_{ij}(v_{ij}^n - t_j^n) > c. \tag{8}$$

$$\text{For each } j, p_j = 0 \text{ if } q \sum_i \gamma_{ij}(v_{ij}^n - t_j^n) < c \text{ for all } n. \tag{9}$$

$$\text{For each } (i, j) \text{ with } \sigma_{ij} > 0, \sum_n t_j^n \leq x_i. \tag{10}$$

$$\text{For each } (i, j), \sum_n v_{ij}^n \leq x_i. \tag{11}$$

The first constraint (4) is the funding requirement that the firm must raise at least I from investors.[8] Substituting the definition of R from (4) into the objective function shows that the problem is equivalent to minimizing the expected contracting costs, the fixed cost Nf plus the expected monitoring cost $c \sum_i \pi_i \sum_j \sigma_{ij} p_j$, subject to the same constraints. Conditions (5) and (6) ensure that the manager chooses a probability measure over reports and that his strategy is rational. Condition (7) says that the probability of monitoring is no greater than 1, and (8) and (9) ensure that investor monitoring is rational. The final constraints (10) and (11) say that the transfers are no larger than the total cash available.

2. OPTIMAL CONTRACTING WITH TWO STATES

This section considers the contracting problem when the project produces only two possible cash flows, which allows an explicit solution and illustrates the basic points. I let π denote the probability of the bad outcome x_1, so

[8] In constraints (4) and (7), I have imposed $\sum_n p_j^n = p_j$. Assuming $\sum_n p_j^n = p_j$ forfeits no generality. Although there are feasible contracts with $\sum_n p_j^n > p_j$ (multiple investors simultaneously investigate report $\hat{x}_j$), the redundant monitoring makes such contracts suboptimal. (But see footnote 7 for a caveat.)

$1 - \pi$ is the probability of the good outcome x_2. Investigating high reports $\hat{x}_2$ just wastes resources, so the optimal contract always has $p_2 = 0$. The example used for illustration has equally likely cash flows of $x_1 = 10$ and $x_2 = 20$ and fixed contracting costs $f = 0.1$. The cost of monitoring is $c = 1.8$ and the probability of success is $q = 0.9$, yielding cost per success $c/q = 2$. The required investment is allowed to vary from 12.9 (Example 1) to 11.9 (Example 2) to 10.9 (Example 3). Since the expected cash flow from the project is 15, the net present value of the project in the absence of contracting costs ranges from 2.1 in Example 1 to 4.1 in Example 3. Contracting costs will be small when the required investment is small because, when less cash needs to be transferred to the investor at project's end, less monitoring is required.

2.1. *Compulsory Monitoring*

Consider the situation when monitoring is contractible, so one need not set the contractual transfers to *induce* investors to monitor. I call this benchmark a "compulsory-monitoring" environment. With compulsory monitoring, the optimal contract has a single investor and truthful reporting. When cash flows are low, the investor gets all the cash, whether or not she monitors: $t = v_{11} = 10$. The investor also gets all the cash if the good type is caught reporting the bad outcome ($v_{21} = 20$), but this does not happen in equilibrium. Since good reports are not investigated, the manager's transfer is t_2 if he tells the truth when cash flows are high; if he were to lie, his expected transfer would be $p_1 q v_{21} + (1 - p_1 q) t_1$.[9] The transfer t_2 from the good type and the monitoring probability p_1 are chosen to simultaneously satisfy the truth-telling constraint $t_2 = p_1 q v_{21} + (1 - p_1 q) t_1$ and the funding constraint (4).

Panel A of Table I illustrates this contract for the examples. In the high-investment/low-profit Example 1, low reports must be audited with probability 0.833. The frequency of monitoring is therefore $\pi p_1 = 0.417$, and expected contracting costs are $f + c\pi p_1 = 0.85$. Examples 2 and 3 have smaller t_2 because less cash needs to be extracted from the manager, due to the smaller required investment. With t_2 smaller, a lower monitoring probability p_1 is enough to induce the good type to report truthfully, so deadweight costs are reduced.

When monitoring is not contractible, investors have monitoring discretion and this contract will not work. When the investor observes a low report, she "knows" the true cash flow is low because the manager reports honestly

[9] When monitoring is contractible, the contract can specify different transfers when monitoring fails and when it is not undertaken at all. In the two-state case, it is optimal to pay out all the cash following report $\hat{x}_1$, so the two transfers are identical and we can use the single label t_1.

TABLE I
Examples of Single-Investor Truth-Telling Contracts

	Example 1	Example 2	Example 3
Required investment I	12.9	11.9	10.9
Average profit	2.1	3.1	4.1
A: Optimal Compulsory-Monitoring Contract			
Monitoring probabilities:			
Probability p_1 when $x = 10$	0.833	0.556	0.278
Probability p_2 when $x = 20$	0	0	0
Monitoring frequency πp_1	0.417	0.278	0.139
Expected monitoring cost $c\pi p_1$	0.75	0.5	0.25
Total expected contracting cost	**0.85**	**0.6**	**0.35**
Transfers without successful monitoring:			
Transfer t_1 when $x = 10$	10	10	10
Transfer t_2 when $x = 20$	17.5	15	12.5
Transfers when monitoring succeeds:			
Transfer v_{11} when $x = 10$	10	10	10
Transfer v_{22} when $x = 20$	N/A	N/A	N/A
B: Discretionary Monitoring—Optimal Single-Investor Truth-Telling Contract			
Monitoring probabilities:			
Probability p_1 when $x = 10$	0.926	0.741	0.556
Probability p_2 when $x = 20$	0	0	0
Monitoring frequency πp_1	0.463	0.370	0.278
Expected monitoring cost $c\pi p_1$	0.833	0.667	0.5
Total expected contracting cost	**0.933**	**0.767**	**0.6**
Transfers without successful monitoring:			
Transfer t_1 when $x = 10$	8	8	8
Transfer t_2 when $x = 20$	18	16	14
Transfers when monitoring succeeds:			
Transfer v_{11} when $x = 10$	10	10	10
Transfer v_{22} when $x = 20$	N/A	≤ 18	≤ 16

Note. The required investment varies across the three examples from 12.9 to 11.9 to 10.9. The other parameters are fixed: the cash flows are $x \in \{10, 20\}$ with equal probabilities, the fixed contracting cost is $f = 0.1$ per investor, the cost of monitoring is $c = 1.8$, and the probability that monitoring succeeds is $q = 0.9$. N/A means that any feasible value will suffice. For Example 1, this contract is optimal under discretionary monitoring; for Examples 2 and 3, the contract can be improved using false reporting or multiple investors.

in equilibrium. If she monitors, she receives a payment of 10 and incurs monitoring cost $c = 1.8$. If she does not monitor, she receives a payment of 10 and incurs no monitoring cost, so she would forgo the audit. The equilibrium crumbles because, if low reports are not investigated, the good type will falsely report low cash flows (reducing his transfer to only 10), and the payments received by the investor will not be large enough to justify the initial investment. The situation is even worse when there are more than two states. Under compulsory monitoring, the investor is sometimes forced to monitor even though her transfer v_{ii} is *strictly smaller* than t_i.[10]

2.2. *Contracts with One Investor and Truthful Reporting*

With discretionary monitoring, the optimal contract may involve multiple investors or false reporting, but we begin by finding the best single-investor contract that produces truthful reporting. The investor will investigate a report only if the contract increases her expected transfer by c. Thus, when the contract induces honest reporting, $q(v_{11} - t_1) \geq c$ is needed to induce monitoring and $q(v_{11} - t_1) = c$ is needed to induce random monitoring. Stated differently, the additional transfer when monitoring succeeds, $v_{11} - t_1$, must be enough to cover the monitoring cost per success, c/q. In the examples, the contract must increase the investor's transfer by 2 when monitoring succeeds, because $c/q = 1.8/0.9 = 2$. The best one can do is set the transfer as high as possible when monitoring succeeds ($v_{11} = 10$) and set $t_1 = v_{11} - c/q = 8$. This reduction in t_1 from 10 to 8 means that the manager sometimes keeps some cash in the bad state. When the investor forgoes monitoring or monitoring fails, the investor gets 8 and the manager keeps 2. Under compulsory monitoring, the efficient arrangement always pays *all* the cash to the investor in the bad state.

The reduction in t_1 requires an increase in p_1, hence an increase in monitoring costs, for two reasons. First, reducing t_1 makes lying more attractive to the manager when cash flows are high, so p_1 must be increased to restore truth-telling incentives. Second, reducing t_1 means that less cash is transferred when cash flows are low; to justify the initial investment, more cash must therefore be transferred when cash flows are high. This means t_2 must increase, which also makes lying more attractive when cash flows are high, so a further increase in p_1 is needed to persuade the manager to report honestly.

Panel B of Table I illustrates the best single-investor truth-telling contract when monitoring is discretionary. One can see that expected contracting costs are always higher than they would be if monitoring were compulsory, due to the higher monitoring probability, p_1. It turns out that the single-investor truth-telling contract is the *optimal* contract in Example 1, where profits are low. But in Examples 2 and 3, there is a more efficient way to

[10] One can show that the reverse incentive problem never arises. One can always write the benchmark contract so that the investor has no ex post incentive to investigate in states with $p_i = 0$. The investor is sometimes *compelled* to monitor when she would rather not, but is never *forbidden* from monitoring when she prefers to monitor. I therefore refer to the benchmark setting as a compulsory-monitoring environment.

induce appropriate monitoring by investors: false reporting or using two investors. With a single-investor truth-telling contract, one cannot achieve full payout in the bad state, and this necessitates increased monitoring. Management misrepresentation and multiple investors are alternative ways to motivate investors to monitor while *achieving* full payout in the bad state, thus mitigating contracting inefficiencies.

2.3. *Contracts with False Reporting*

One way to achieve full payout in the bad state and still induce monitoring is to have the manager lie occasionally. He sometimes reports low cash flows when they are actually high and, when a false report is exposed, the investor gets all the cash. Table II illustrates the best misrepresentation contract for the examples. Notice that the contract has full payout in the bad state: $t_1 = v_{11} = 10$, as in Table IA. Even though v_{11} is no larger than t_1, the investor is willing to monitor because the good type lies 25% of the time ($\sigma_{22} = 0.75$) and the investor gets all the cash when a false report is uncovered ($v_{21} = 20$). When the investor receives a report that the cash flow is 10, the probability that it is really 10 is

$$\gamma_{11} = \frac{\pi\sigma_{11}}{\pi\sigma_{11} + (1-\pi)\sigma_{21}} = \frac{0.5(1)}{0.5(1) + 0.5(0.25)} = 0.8;$$

there is a 20% chance the report came from an impostor with lots of cash. This provides an incentive to monitor, because monitoring increases the expected transfer by

$$q[\gamma_{11}(v_{11} - t_1) + \gamma_{21}(v_{21} - t_1)] = 0.9[0.8(10 - 10) + 0.2(20 - 10)] = 1.8,$$

enough to cover the monitoring cost. Equivalently, the incremental expected payoff when monitoring succeeds, $0.2(20 - 10) = 2$, is enough to cover the cost per success, $c/q = 2$.

Comparing expected contracting costs in Table II with those in Table IB, one sees that the misrepresentation contract is better in the case of high profits, Example 3. In fact, the misrepresentation contract is the *optimal* contract for this example. We shall see later that this is true even with an arbitrary number of states—there are always efficiency gains from false reporting when expected profits are high.

The advantage of a misrepresentation contract is that achieving full payout in the bad state allows us to reduce p_1, for the reasons explained in Section 2.2, which lowers the cost incurred monitoring the bad type. The disadvantage is that resources are now expended monitoring the *good* type, who now sends some low reports—the monitoring frequency is $\pi p_1 +$

TABLE II
Examples of Misrepresentation Contracts

	Example 1	Example 2	Example 3
Required investment I	12.9	11.9	10.9
Average profit	2.1	3.1	4.1
Reporting strategies:			
Truth telling prob. σ_{11} when $x = 10$	1	1	1
Truth telling prob. σ_{22} when $x = 20$	0.75	0.75	0.75
Monitoring probabilities:			
Probability p_1 when $\hat{x} = 10$	0.889	0.593	0.296
Probability p_2 when $\hat{x} = 20$	0	0	0
Monitoring frequency $[\pi + (1 - \pi)\sigma_{21}]p_1$	0.556	0.370	0.185
Exp. monitoring cost $c[\pi + (1 - \pi)\sigma_{21}]p_1$	1	0.667	0.333
Total expected contracting cost	**1.1**	**0.767**	**0.433**
Transfers without successful monitoring:			
Transfer t_1 when $\hat{x} = 10$	10	10	10
Transfer t_2 when $\hat{x} = 20$	18	15.33	12.67
Transfers when monitoring succeeds:			
Transfer v_{11} when $x = 10$ and $\hat{x} = 10$	10	10	10
Transfer v_{12} when $x = 10$ and $\hat{x} = 20$	N/A	N/A	N/A
Transfer v_{21} when $x = 20$ and $\hat{x} = 10$	20	20	20
Transfer v_{22} when $x = 20$ and $\hat{x} = 20$	N/A	$\leq$17.33	$\leq$14.67
Conditional state probabilities:			
Prob. γ_{11} that $x = 10$, given $\hat{x} = 10$	0.8	0.8	0.8
Prob. γ_{22} that $x = 20$, given $\hat{x} = 20$	1	1	1

Note. This continues the examples from Table I by exhibiting the best contract that involves false reporting by the manager. Comparing expected contracting costs with the single-investor truth-telling contract in Table IB, one can see that the misrepresentation contract is worse in Example 1, equivalent in Example 2 and better in Example 3. This is the optimal contract for Example 3.

$(1 - \pi)\sigma_{21}p_1 > \pi p_1$. For this example, the benefits of misrepresentation outweigh the costs when expected profits are high, but not when they are low.

Notice that a manager sending a false report receives a payoff of $x_2 - t_1 = 10$ when this is not discovered, but is "punished" by having his entire surplus taken when the investor discovers that he sent a false report. In Example 3 at least, the fact that the good manager type sends some false reports *improves* efficiency ex ante, yet that manager suffers when such reports are uncovered. This punishment is perhaps ironic, but the whole reason that misrepresentation helps is that the investor *does* take a lot of cash when she uncovers a false report, providing the incentive to monitor.

2.4. *Contracts with Two Investors*

When monitoring is discretionary, another possibility for improving efficiency is to contract with two investors rather than one. Given our assumptions, a two-investor contract is inefficient when monitoring is compulsory because it merely duplicates the fixed contracting cost f.[11] But when investors must be induced to monitor, adding a second investor provides flexibility in contract design, so we can achieve full payout in the bad state even when the manager reports truthfully. Each investor's monitoring incentives depend on her individual transfer, but the manager's reporting incentives depend on the total transfer. With two investors, we can give one investor the appropriate incentive to monitor and adjust the total transfer by changing the payment to the other investor (as long as we do not give her a strict incentive to monitor). The result is lower monitoring costs than with a single-investor contract, but higher fixed costs of contracting.

With only two states, a two-investor misrepresentation contract is never optimal—the only multiple-investor contract that can be optimal is a two-investor truth-telling contract.[12] Such a contract is illustrated in Table III. The superscripts on the monitoring probabilities p_i^n and transfers t_i^n and v_{ii}^n refer to the investor. Notice that the monitoring frequency is lower than under the other two contracts (Table IB and Table II). This contract gives Investor 1 the incentive to investigate low reports. When a low report is made and Investor 1 does not produce proof of x_1, she receives 8 and Investor 2 receives 2, for a total payout of 10; when she monitors successfully, she receives 10 and Investor 2 receives nothing, so the total payout is again 10. Investor 1 is willing to monitor because increasing the transfer from 8 to 10 covers her monitoring cost per success, $c/q = 2$. Investor 2 has a strict incentive to avoid monitoring since monitoring is costly and successful monitoring actually *reduces* her transfer by 2. When Investor 1 monitors, it is Investor 2, not the manager, who ends up bearing the monitoring cost by having her expected transfer reduced. Monitoring increases Investor 1's expected transfer from 8 to 9.8 and decreases Investor 2's

[11] The assumption that the fixed contracting cost is linear in the number of investors is completely unimportant for the substance of the results. One could allow fixed contracting costs to be any increasing function of the number of investors.

[12] This is because, with only two states, adding a second investor eviscerates the investor incentive constraints, allowing monitoring costs just as low as one could get with compulsory monitoring (with fixed cost $2f$ rather than f). A misrepresentation contract generates some monitoring of the good type, which we know is inefficient with compulsory monitoring, so multiple-investor contracts with misrepresentation are inefficient when $S = 2$.

TABLE III
Examples of Two-Investor Truth-Telling Contracts

	Example 1	Example 2	Example 3
Required investment I	12.9	11.9	10.9
Average profit	2.1	3.1	4.1
Monitoring probabilities:			
Prob. p_1^1 of Investor 1 for report $\hat{x} = 10$	0.861	0.583	0.306
Prob. p_2^1 of Investor 1 for report $\hat{x} = 20$	0	0	0
Probabilities p_i^2 for Investor 2	0	0	0
Monitoring frequency πp_1	0.431	0.292	0.153
Expected monitoring cost $c\pi p_1$	0.775	0.525	0.275
Total expected contracting cost	**0.975**	**0.725**	**0.475**
Transfers without successful monitoring:			
Transfer t_1^1 to Investor 1 when $x = 10$	8	8	8
Transfer t_1^2 to Investor 2 when $x = 10$	2	2	2
Transfer t_2^1 to Investor 1 when $x = 20$	8	8	8
Transfer t_2^2 to Investor 2 when $x = 20$	9.75	7.25	4.75
Transfers when monitoring succeeds:			
Transfer v_{11}^1 to Investor 1 when $x = 10$	10	10	10
Transfer v_{11}^2 to Investor 2 when $x = 10$	0	0	0
Transfer v_{22}^1 to Investor 1 when $x = 20$	≤ 10	≤ 10	≤ 10
Transfer v_{22}^2 to Investor 2 when $x = 20$	≤ 11.75	≤ 9.25	≤ 6.75

Note. This continues the examples from Table I by exhibiting the best two-investor contract. Comparing the expected contracting costs with the single-investor truth-telling contract in Table IB and the misrepresentation contract in Table II, one can see that the two-investor contract is optimal for Example 2, but not for Examples 1 and 3.

expected transfer from 2 to 0.2. This conflict between securityholders allows more efficient monitoring by achieving full payout in the bad state.

The monitoring probability for bad reports (p_1) and the total transfer when cash flows are high ($t_2^1 + t_2^2$) are chosen to satisfy the good type's truth-telling constraint and the funding constraint. The optimal division of the transfer between the two investors is not unique. The contract in Table III is chosen so that Investor 1 has a debt-like contract—she is "promised" a constant payment of $t_1^1 = t_2^1 = 8$, but she is paid a "bonus" when she monitors successfully, and this covers her monitoring costs. Investor 2 gets a small transfer when cash flows are low (0 or 2, depending on whether Investor 1 monitors successfully) and a larger transfer when cash flows are high, so she holds an equity-like security.

Sometimes the savings in monitoring costs from using two investors are enough to offset the additional fixed contracting costs, and sometimes they

TABLE IV
Summary of Examples

	Example 1	Example 2	Example 3
Required investment I	12.9	11.9	10.9
Average profit	2.1	3.1	4.1
Expected contracting cost:			
Compulsory-monitoring benchmark	0.85	0.6	0.35
Single-investor truth-telling contract	**0.933**	0.767	0.6
Single-investor misrepresentation contract	1.1	0.767	**0.433**
Two-investor truth-telling contract	0.975	**0.725**	0.475

Note. This summarizes the examples from Tables I, II, and III. It gives the expected contracting cost (fixed cost plus expected monitoring cost) for the three types of contracts and for the benchmark case where the manager can be costlessly compelled to monitor. The expected contracting cost for the optimal contract is in boldface.

are not. The two-investor contract is the optimal contract in Example 2, but it is suboptimal in Examples 1 and 3. The next subsection shows that this is true in general—when all three types of contracts are optimal for some expected profit level, a two-investor contract is optimal in the intermediate range of expected profits. Table IV summarizes the examples, listing the expected contracting cost for the three types of contracts and for the compulsory-monitoring benchmark.

2.5. *Results for the Two-State Case*

The two propositions below summarize the results when there are two possible cash flows.

PROPOSITION 1. *When $S = 2$, the optimal contract takes one of the following forms:*

1. (*Single-investor truth-telling contract*) *The contract has $N = 1$, has reporting strategies $\sigma_{11} = \sigma_{22} = 1$, has monitoring probabilities $p_1 = (I + f - x_1 + c/q)/[(1 - \pi)q(x_2 - x_1 + c/q)]$ and $p_2 = 0$, and has transfers $t_1 = x_1 - c/q$, $v_{11} = x_1$, and $t_2 = p_1 q x_2 + (1 - p_1 q)(x_1 - c/q)$. Expected contracting costs are $f + c\,\pi(I + f - x_1 + c/q)/[(1 - \pi)q(x_2 - x_1 + c/q)]$.*

2. (*Two-investor truth-telling contract*) *The contract has $N = 2$, has reporting strategies $\sigma_{11} = \sigma_{22} = 1$, has monitoring probabilities $p_1 = (I + 2f - x_1)/[(1 - \pi)q(x_2 - x_1) - \pi c]$ and $p_2 = 0$, and only one investor monitors. The transfers satisfy $t_1^1 + t_1^2 = v_{11}^1 + v_{11}^2 = x_1$, $t_2^1 + t_2^2 = p_1 q x_2 + (1 - p_1 q)x_1$, and $v_{21}^1 + v_{21}^2 = x_2$. If Investor 1 is the one who monitors, then $v_{11}^1 - t_1^1 = c/q$ and $v_{11}^2 - t_1^2 = -c/q$. Expected contracting costs are $2f + c\pi(I + 2f - x_1)/[(1 - \pi)q(x_2 - x_1) - \pi c]$.*

3. (*Misrepresentation contract*) *The contract has* $N = 1$, *has reporting strategies* $\sigma_{11} = 1$ *and* $\sigma_{22} = [(1 - \pi)(x_2 - x_1) - c/q]/[(1 - \pi)(x_2 - x_1 - c/q)]$, *has monitoring probabilities* $p_1 = (I + f - x_1)(x_2 - x_1 - c/q)/[(x_2 - x_1)[(1 - \pi)q(x_2 - x_1) - c]]$ *and* $p_2 = 0$, *and has transfers* $t_1 = v_{11} = x_1$, $t_2 = p_1 q x_2 + (1 - p_1 q)x_1$, *and* $v_{21} = x_2$. *Expected contracting costs are* $f + c\pi(I + f - x_1)/[(1 - \pi)q(x_2 - x_1) - c]$.

Notice that when the required investment is small (I close to $x_1 - f$), the misrepresentation contract has expected contracting costs close to f, while the other two contracts have costs bounded away from f. Expected monitoring costs approach zero only for the misrepresentation contract, making it clearly optimal for the most profitable projects. Notice also that expected contracting costs are linear in I for each of the three contracts. Thus, the rate at which expected contracting costs change as project size changes, the marginal cost of capital, is constant for each contract. The marginal cost of capital is highest for the misrepresentation contract and is lowest for the single-investor truth-telling contract. As the required investment varies over the range allowed in (1), the "typical" pattern is that the misrepresentation contract is optimal for small investments, the two-investor truth-telling contract is optimal over an intermediate range and the single-investor truth-telling contract is optimal for large investments. However, only the misrepresentation contract is always optimal over *some* range of investment levels, regardless of the parameters.[13] In general, if truth-telling contracts are used, it is always over a range of large investment levels, i.e., low expected profits. High expected profits favor the misrepresentation contract.

The most interesting case, in which all three contracts are optimal over some range, is illustrated in Figure 1, which plots expected contracting costs as a function of I for the previous example. The misrepresentation contract (the dotted line) is optimal on the left interval, where profits are high, and the single-investor truth-telling contract (the solid line) is optimal on the right interval, where profits are low. The two-investor contract (the dashed line) is optimal in between. The dot-dash line is expected contracting costs under compulsory monitoring, so the gap between this line and the best discretionary-monitoring contract is the lost wealth due to noncontractible monitoring. The wealth loss is greatest in the intermediate range and approaches zero as profits get very large or very small.

One can also examine how project risk affects contract choice. With only two cash flows, a mean-preserving spread means that the bad state gets worse

[13] The two-investor contract is never used if f is large, the single-investor truth-telling contract is never used if f is small or π and q are large, and neither truth-telling contract is used if f, π and q are all large. In this last case, the misrepresentation contract is optimal for *all* investment levels satisfying (1).

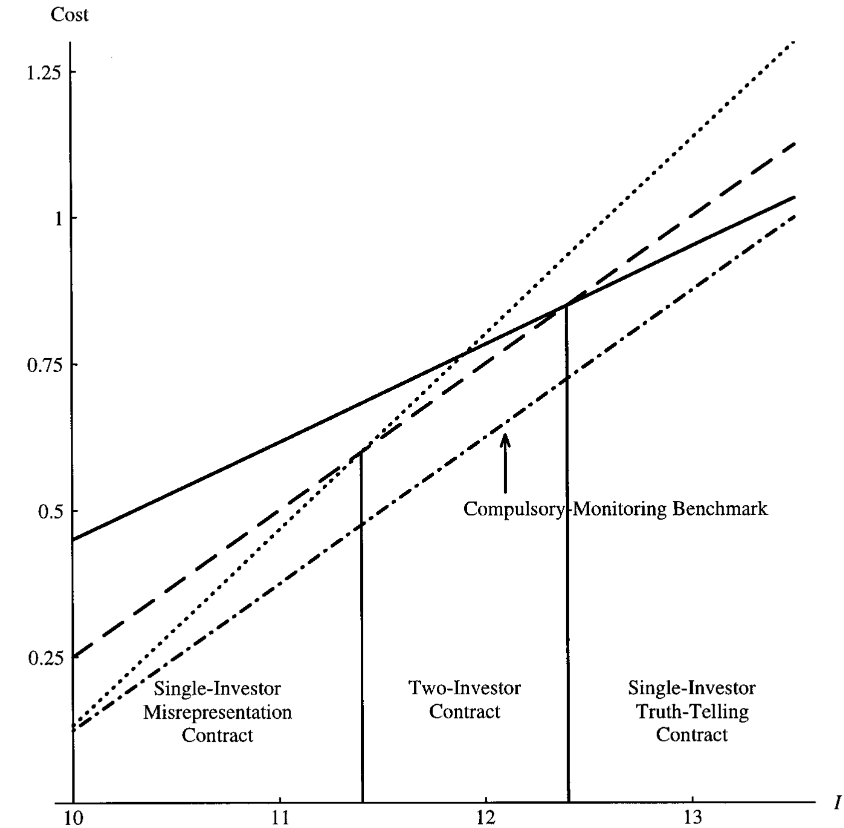

FIG. 1. Expected contracting cost as a function of required investment. The figure plots the expected contracting cost for an example that has two equally likely states with cash flow either $x_1 = 10$ or $x_2 = 20$, fixed contracting cost $f = 0.1$, monitoring cost $c = 1.8$ and probability $q = 0.9$ that monitoring succeeds. Cost is plotted as a function of the project's required investment I. The solid line is the single-investor truth-telling contract; the dotted line is the single-investor contract with false reporting; the dashed line is the contract with multiple investors; and the dot-dash line is the benchmark case of compulsory monitoring.

and the good state gets better, with $E[\tilde{x}] = \pi x_1 + (1 - \pi)x_2$ unchanged. A misrepresentation contract is optimal if the cash flow distribution is safe enough, i.e., x_1 is above some cutoff value. Furthermore, as long as the required investment is not too small, a truth-telling contract is optimal if the distribution is risky enough.[14] These results are stated in the next proposition.

[14] For some parameterizations, misrepresentation is optimal no matter how risky the distribution. Thus, the additional restriction on investment size in Proposition 2 is needed to ensure that truth-telling is sometimes optimal.

PROPOSITION 2. *Suppose* $S = 2$, *and let* $\underline{I} < I < \bar{I}$ *denote the allowable investment levels given by* (1). *There are critical values* I_0 *and* I_1 *with* $\underline{I} < I_0 \leq I_1 \leq \bar{I}$ *such that*

1. *If* $\underline{I} < I < I_0$, *the optimal contract is a misrepresentation contract.*
2. *If* $I_0 < I < I_1$, *the optimal contract is a two-investor truth-telling contract.*
3. *If* $I_1 < I < \bar{I}$, *the optimal contract is a single-investor truth-telling contract.*

Misrepresentation is efficient when the risk of the project cash flows, in the sense of Rothschild and Stiglitz (1970), *is low enough. Assuming* $I > E[\tilde{x}]/(2 - \pi) + (c/q)(1 - \pi)/(2 - \pi)) - f$, *truth-telling is efficient when the risk of the project cash flows is high enough.*

Low-variance distributions favor misrepresentation for the same reason that small investments favor misrepresentation: p_1 is lower. The drawback of a misrepresentation contract is that resources are squandered monitoring the good type, but very little is squandered when p_1 is small.

These results suggest that we should see more misrepresentation when, at the contracting date, the firm expects high profits and there is relatively little uncertainty about the cash flows. Misrepresentation is less likely when expected profits are low and cash flows are volatile. We should see simple capital structures (in the model, a single class of securityholders) when cash flows are stable and expected profits are high; more complex financing structures require lower profitability and higher volatility.

3. OPTIMAL CONTRACTING WITH MANY STATES

Section 2 provided a complete characterization of optimal contracts with two states of the world. This section shows that the basic results extend to an arbitrary number of states—both misrepresentation and conflicting interests among securityholders can promote efficiency in a world of discretionary monitoring.

The first result shows that, for the general case of S states, misrepresentation is optimal when the required investment is small. The proposition is proved by constructing a misrepresentation contract that does better than any truth-telling contract when I is small.

PROPOSITION 3. *When* I *is small* (*close enough to* $x_1 - f$), *the solution to Problem* 1 *involves false reporting:* $\sigma_{ii} < 1$ *for some* i.

The second result of this section shows that, for the general case of

S states, securityholder conflict remains optimal when multiple investors finance the firm. Whenever a multiple-investor truth-telling contract is optimal, it utilizes two investors. If Investor 1 investigates report $\hat{x}_i$, her expected transfer must increase by c: $q(v_{ii}^1 - t_i^1) \geq c$. The monitoring investor's transfer could be increased by reducing the manager's residual but, like the two-state case, the optimal contract reduces the other investor's transfer instead. Furthermore, unlike the two-state case, the other investor's transfer is sometimes reduced by *more than* the monitoring cost, and the manager's residual actually increases: $q(v_{ii}^2 - t_i^2) \leq -c$, and the inequality is sometimes strict. If Investor 1 monitors, Investor 2 bears the monitoring cost, and possibly an additional penalty. Placing the securityholders in conflict over monitoring policy produces efficient monitoring.

PROPOSITION 4. *Suppose the solution to Problem* 1 *has multiple investors and truthful reporting, and report $\hat{x}_i$ induces monitoring: $p_i > 0$. Then the optimal contract has two investors, only one investor* (*e.g., Investor 1*) *investigates report $\hat{x}_i$, and the other investor bears the entire cost of monitoring: $q(v_{ii}^2 - t_i^2) \leq -c$.*

The result can also be stated as $v_{ii}^1 + v_{ii}^2 \leq t_i^1 + t_i^2$; given that $v_{ii}^1 - t_i^1 \geq c/q$ to induce Investor 1 to monitor, this requires $v_{ii}^2 - t_i^2 \leq t_i^1 - v_{ii}^1 \leq -c/q$. The reason that $v_{ii}^1 + v_{ii}^2$ cannot be greater than $t_i^1 + t_i^2$ is that such a contract provides inefficient truth-telling incentives to the manager. Slightly higher $t_i^1 + t_i^2$ and lower $v_{ii}^1 + v_{ii}^2$ could generate the same expected revenue to investors and make it less tempting to send a false report $\hat{x}_i$. The resulting slack in the truth-telling constraint could then be exploited to reduce monitoring probabilities.

4. RENEGOTIATION AND MISREPRESENTATION

The previous sections abandoned the standard assumption that monitoring is contractible, but maintained the standard assumption that the parties commit to the contractual transfer schedule, $\{t_j^n, v_{ij}^n\}$. This section investigates what happens if we make contracting even more difficult by assuming that the parties can, at some cost, renegotiate the transfer schedule. The approach is to allow a simple form of renegotiation and examine whether a contract can survive when renegotiation is allowed.

We shall see that allowing renegotiation reinforces the message that equilibrium misrepresentation can be helpful. Absent renegotiation, a truth-telling contract is optimal for some parameters and a misrepresentation contract is optimal for others. However, when renegotiation is possible and costs of renegotiation are small, the optimal contract is never a truth-telling contract (anywhere in the parameter space). The only reports $\hat{x}_j$ that can be taken at face value are reports that have monitoring probability p_j below some cutoff. Investors must be suspicious of any report with p_j above this cutoff, since all such reports have $\gamma_{jj} < 1$. As costs of renegotiation go to zero, misrepresentation becomes rampant: this cutoff probability goes to zero, so all reports that elicit monitoring are tainted by false reporting.

The result that misrepresentation can be helpful in preventing renegotiation is similar to the results of Hart and Tirole (1988) on sale versus rental of a durable good, Dewatripont (1989) on employment, Laffont and Tirole (1990) on military procurement, and Fudenberg and Tirole (1990) on moral hazard. These papers all find that keeping the uninformed party relatively uninformed—e.g., by having the informed party play a mixed strategy—can help ensure renegotiation-proofness.

To address renegotiation, I restrict attention to single-investor contracts for the formal analysis. Multiple-investor contracts are discussed briefly at the end of the section. Since the entrepreneur/manager has the bargaining power at the initial contracting date (he obtains the surplus), I assume that he also has the bargaining power at the renegotiation date. A single round of renegotiation is allowed as follows. After the manager sends his cash flow report, and before the investor has made her monitoring decision, the manager decides whether to offer revised contract terms and the investor decides whether to accept the new terms, if offered.[15] I use an ^{a} to denote the alternative contract that is offered. If a new contract is proffered and accepted, the contract is revised at (exogenous) cost $r_M > 0$ to the manager and $r_I > 0$ to the investor, and events proceed as usual with the transfers determined by the new contract.[16] If the manager chooses not to offer a new contract or if the investor rejects the new terms, renegotiation ends and events proceed under the terms of the initial contract. A contract is renegotiation-proof given r_M and r_I if no manager can offer a contract at

[15] In the literature on renegotiation under asymmetric information, the most common assumption is that the uninformed party proposes the renegotiation, since this permits one to ignore the possibility that the offer signals information; exceptions are Gale and Hellwig (1989) and Maskin and Tirole (1992). It seems natural in my setting to assume that the manager (the informed party) proposes the renegotiation. Management typically proposes exchange offers, recapitalizations, and other such restructurings, and management has the exclusive right (for a period of time) to propose a plan of reorganization under Chapter 11 of the U.S. bankruptcy code.

[16] Formally, I treat r_M as a nonpecuniary cost borne by the manager. The advantage of this assumption is that renegotiation does not affect the maximum possible transfer to the investor. The alternative assumption that r_M is a cost paid by the firm, thus reducing any residual, would shrink the maximum possible transfer to the investor from x_j to $x_j - r_M$, and this would introduce complications without changing the substance of the results. See Rubinstein and Wolinsky (1992) for a discussion of costly renegotiation.

the renegotiation stage that is acceptable to the investor and makes the manager better off.[17]

Consider the situation when a single-investor truth-telling contract is optimal absent renegotiation. The maximum monitoring probability (if the contract is designed efficiently) is $p_1 > 0$. The contract has $v_{11} = t_1 + c/q$ to induce investigation of reports $\hat{x}_1$; however, such investigations are a waste of resources ex post, as the manager's report reveals the state. It is easy to see that if renegotiation costs are small enough—namely, $r_M + r_I < p_1 c$—the manager can improve his lot by offering the investor a new contract. Ex ante welfare is reduced because the contract cannot survive. For instance, after reporting $\hat{x}_1$, the manager could offer to set $v_{11}^a = 0$ and $t_1^a = t_1 + r_I$. Accepting this proposal gives the investor the same expected payoff as the incumbent contract (taking account of renegotiation costs), and the manager's payoff increases under the proposed contract because no resources are spent on monitoring. Because p_1 is the largest p_i, the condition $r_M + r_I \geq p_1 c$ is necessary and sufficient for renegotiation-proofness.

PROPOSITION 5. *If a single-investor truth-telling contract is optimal absent renegotiation, the contract remains optimal when renegotiation is possible if and only if $r_M + r_I \geq p_1 c$, where p_1 is the probability that report $\hat{x}_1$ is investigated. Any renegotiation-proof single-investor truth-telling contract has $p_i \leq (r_M + r_I)/c$ for all i.*

When the parties cannot commit to the transfer terms and costs of renegotiation are small, this time-inconsistency problem would seem almost insurmountable. With each p_i bounded by $(r_M + r_I)/c$, only very small monitoring probabilities can be supported because more frequent monitoring would be undermined by opportunistic renegotiation. This means that only the most profitable projects can be financed—the investor is willing to invest very little ex ante because very little revenue can be extracted from the manager ex post. One might therefore expect the parties to take actions that make renegotiation more difficult, i.e., that raise the renegotiation costs r_M and r_I. One example is issuing the security publicly to widely dispersed investors rather that issuing privately to a few investors, with the intention of making renegotiation more difficult (Gertner and Scharfstein, 1991). If the parties can legally forbid themselves from renegotiating the initial contract, even when it is mutually agreeable to do so ex post, this

[17] One can think of the costs r_M and r_I as the administrative costs of rewriting the contract or perhaps the costs of haggling over the terms of the contract. Common practice in the renegotiation literature is to assume that renegotiation costs are zero. In the case of renegotiating the terms of a security issue, costs are nontrivial, certainly not zero, so it seems reasonable to treat them as positive. Nevertheless, I also consider what happens as these costs get small, approaching the case of costless renegotiation.

would solve the problem. One can view this as setting r_M and r_I very high; $r_M + r_I$ might be the cost of legal counsel that is ingenious enough to legally circumvent the previous prohibition, or the cost the parties must incur to prevent themselves from illegally circumventing the contract through a side agreement.

However, there might be a more efficient solution than artificially raising renegotiation costs. In fact, even with very low renegotiation costs, there might be no problem to begin with. The previous result applies to *truth-telling* contracts, and misrepresentation can once again be beneficial. Not only can false reporting improve efficiency by providing effective monitoring incentives, but it can also improve efficiency by making contracts immune to renegotiation. In the two-state case, for instance, the misrepresentation contract of Proposition 1 is *always* renegotiation-proof. Therefore, when the parameters are such that the misrepresentation contract is optimal absent renegotiation, there is no welfare loss when renegotiation is possible, even if renegotiation costs are very low.

PROPOSITION 6. *Suppose $S = 2$. The misrepresentation contract of Proposition* 1 *is renegotiation-proof for all r_M and r_I.*

To understand this result, recall that under the misrepresentation contract of Proposition 1, the good type sometimes falsely reports $\hat{x}_1$. Reports $\hat{x}_2$ do not prompt monitoring, so renegotiation is feasible only after a bad report. Following a bad report, the investor gets the maximum possible cash, conditional on the information: $v_{21} = x_2$ if a false report is uncovered, and $t_1 = v_{11} = x_1$ if not. Because t_1 is at its maximum value, the manager cannot offer to increase it to induce the investor to forgo monitoring.[18] Therefore, there is no mutually beneficial renegotiation strategy—the contract is renegotiation-proof even if r_M and r_I are tiny.

Proposition 5 gives some sense of the extent of misrepresentation in the case of an arbitrary number of states. An immediate corollary of Proposition 5 is that, when renegotiation is possible and renegotiation costs are small, the only reports that can be believed are those that elicit little monitoring. Any report that has $p_j > (r_M + r_I)/c$ must be viewed with suspicion, as there is some probability that the report is false: $\gamma_{jj} < 1$. When renegotiation costs are very close to zero, then, roughly speaking, only reports that elicit no monitoring are credible; all reports that are investigated with positive probability are tainted by false reporting.

When one allows multiple investors and renegotiation, complications arise that are absent with a single investor. One advantage of issuing more than one type of security is that any subsequent renegotiation involving

[18] If the manager reduced v_{11} or v_{21} to eliminate incentives to monitor, the investor would be better off simply rejecting the offer.

three or more parties will presumably be more involved and costly, and it will be more difficult to construct a new contract that is acceptable to all the parties. Thus, using multiple securities might be an effective way to impede renegotiation, allowing a more efficient ex ante contract to survive. On the other hand, a contract involving three or more parties might be more susceptible to renegotiation because of coalitional issues. For example, if the manager contracts separately with two investors as in Table III, Investor 1's decision to monitor hurts Investor 2 by reducing her transfer with probability q. These two investors could sign a "side deal" in which Investor 2 pays some amount $y \in (0, 1.8)$ to Investor 1 when the manager reports $\hat{x}_1$ and Investor 1 does *not* produce evidence of the true state.[19] This makes Investor 1 averse to monitoring rather than indifferent and, ex post, benefits both investors due to the monitoring cost savings. If such deals are possible, the contracts in Table III are infeasible. In Example 2, where the two-investor contract is optimal absent renegotiation, a less efficient contract must be used.

5. RELATED LITERATURE

Previous studies by Graetz, Reinganum, and Wilde (1986), Reinganum and Wilde (1986), Beck and Jung (1989), Baiman, Evans, and Noel (1987), Melumad and Mookherjee (1989), and Lacker (1989) have considered discretionary verification in Townsend-like settings. The first three papers are quite different from this model in that they examine exogenous transfer schedules rather than *optimal* contracts; the last three papers are closer in spirit to this one.

Melumad and Mookherjee (1989) search for an income tax scheme that maximizes a social welfare function, where taxes collected are used to provide a public good. They assume the I.R.S. can commit to aggregate dimensions of audit policy, like total audit costs and total collections. They show that in this "limited-commitment" setting, the "full-commitment" outcome (with truthful reporting) can be achieved if the I.R.S. delegates auditing responsibility to an agent. This type of delegation based on aggregate dimensions of audit policy is less reasonable in the current setting where a particular investor finances a particular firm, so I analyze the "no-commitment" case which Melumad and Mookherjee (1989) do not pursue.[20]

[19] See Tirole (1986), Kofman and Lawarée (1993), and Laffont and Martimort (1997), e.g., for more on this type of collusion.

[20] Melumad and Mookherjee (1989) also assume risk averse taxpayers; the manager here is risk neutral, so risk-sharing considerations play no role in these results. Naturally, the question of multiple investors does not arise in their (income tax) setting, but it does in my setting.

Baiman *et al.* (1987) is similar in that the principal commits to a compensation scheme for an independent auditor. However, their auditor can choose to file fraudulent reports by reporting an outcome to the principal without carrying out the audit. The only reason an auditor incurs the audit cost and makes useful reports is that there is an exogenous probability that the true outcome becomes public information. Audits will *never* be conducted if the agent in their model reports truthfully, because the auditor would simply mimic the agent's reports and avoid the audit costs, so one is "forced" to use false reporting by the agent to induce the auditor to carry out the audit. In this model, by contrast, the "auditor" will monitor a manager who reports truthfully if the transfers are set to provide proper incentives—false reporting is not *necessary* to induce monitoring. I show that, nevertheless, false reporting is often socially beneficial.

The model closest to this one is in Lacker (1989). One difference that aids tractability is that I abstract from risk-sharing concerns by assuming universal risk neutrality. This allows me to make headway on the question of misrepresentation, providing conditions under which false reporting produces Pareto improvements; Lacker (1989) recognizes that an optimal contract may require misrepresentation, but does not pursue the matter.[21] The possibility of multiple investors is not considered in Lacker (1989), but I show that multiple investors can produce Pareto gains.

Bhattacharya and Krishnan (1995) pursue a different angle on the question of management misrepresentation. They analyze a "cheap talk" setting in which the cash flow is unverifiable, so a manager's reports can never be proved wrong, and show that managers might nevertheless tell the truth in equilibrium.

Finally, other work has shown that one cannot apply the Revelation Principle when there are exogenous restrictions on the reports that can be sent. One trivial example is when the number of possible reports is smaller than the number of types, so "truthful" reporting by each type is impossible. A nontrivial case is when the set of possible reports varies across types. Unless these report sets satisfy a fairly restrictive condition [the Nested Range Condition of Green and Laffont (1986)], the Revelation Principle does not apply.[22] This factor is absent here—the gains from false reporting are entirely due to monitoring discretion, not due to exogenous restrictions on feasible reports.

6. CONCLUSION

I analyze financial contracting when it is costly for investors to acquire management's private information, and investors *choose* whether to incur this cost after the manager reports the firm's performance—investors have

[21] He states on page 19, "It would seem impossible to eliminate mixed strategies for agent *a* in general."

[22] See Arya *et al.* (1992) for an application.

discretion about monitoring because it is noncontractible. When the parties can commit not to renegotiate the contract, it is sometimes optimal to write the firm's financial contracts in a way that induces the manager to send false reports to investors. This can be an efficient way to get investors to monitor management, because investors can be given large transfers when a false report is exposed. It is also sometimes efficient to issue more than one security, even though this entails higher fixed costs. The second security adds flexibility in providing monitoring incentives to investors. When one investor chooses to monitor, the other investor bears the cost through a lower expected transfer—securityholder conflict is efficient.

When renegotiation is possible and renegotiation costs are small, a truth-telling contract cannot survive, but contracts involving misrepresentation can survive. Thus, not only is misrepresentation an effective means of providing monitoring incentives, but it also helps make contracts immune to renegotiation.

APPENDIX: PROOFS

Proof of Proposition 1

The proof is straightforward, but long and tedious. Here is an outline; the details are available from the author.

To find the best single-investor truth-telling contract, simplify Problem 1 by setting $S = 2$, $N = 1$, $\sigma_{ii} = \gamma_{ii} = 1$, and $\sigma_{ij} = \gamma_{ij} = 0$ $(i \neq j)$. Straightforward analysis of this simplified problem produces the contract in part 1 of the proposition.

A multiple-investor contract has fixed contracting cost at least equal to $2f$. Now, suppose we eliminate the investor incentive constraints (8) and (9) from Problem 1 and find the best contract satisfying this relaxed problem, assuming the fixed contracting cost is $2f$. No multiple-investor contract can have lower expected contracting costs than this "target," and any contract with more than two investors is strictly worse since it has monitoring costs at least as large and has larger fixed costs. With the investor incentive constraints eliminated, one can restrict attention to single-investor truth-telling contracts, so this target can be found by simplifying Problem 1 as follows. Replace the Nf in (4) with $2f$, set $S = 2$, $N = \sigma_{ii} = \gamma_{ii} = 1$, and $\sigma_{ij} = \gamma_{ij} = 0$ $(i \neq j)$, and eliminate constraints (8) and (9). Solving this problem is straightforward, and one can show that the contract in part 2 of the proposition has expected contracting costs equal to this target.

The most involved part of the proof is showing that the only contract that induces false reporting which can possibly be optimal is the contract in part 3 of the proposition. The steps are the following:

1. No optimal contract has $\sigma_{11} = \sigma_{21} = 1$. (Eliminate complete pooling.)

2. No optimal contract has $\sigma_{22} = 1$ and $0 < \sigma_{11} < 1$. (Eliminate false reporting by the bad type alone.)

3. No optimal contract has $0 < \sigma_{11} < 1$ and $0 < \sigma_{22} < 1$. (Eliminate false reporting by both types.)

4. $p_2 = 0$ and $0 < p_1 < 1$.

5. Since $0 < \sigma_{22} < 1$ and $p_2 = 0$, we must have $t_2 = p_1 q v_{21} + (1 - p_1 q)t_1$, which allows us to eliminate t_2 as a choice variable.

6. Since $0 < p_1 < 1$, (8) and (9) require $q(\gamma_{11}v_{11} + \gamma_{21}v_{21} - t_1) = c$, which can be written $\pi(v_{11} - t_1 - c/q) + (1 - \pi)(1 - \sigma_{22})(v_{21} - t_1 - c/q) = 0$.

7. $v_{11} < t_1 + c/q < v_{21}$.

8. $v_{11} = x_1$ and $v_{21} = x_2$.

9. The funding constraint (4) binds.

10. Using the funding constraint and the investor incentive constraint, solve simultaneously for σ_{22} and p_1 as functions of t_1. This reduces the problem to choosing t_1.

11. $t_1 = x_1$ is the only choice that is ever optimal.

Proof of Proposition 2

To prove the result about investment levels, begin by defining

$$\hat{I}_0 = \min\left\{x_1 - f + \frac{(1-\pi)(x_2 - x_1) - c/q}{2 - \pi}, x_1 - f \right.$$
$$\left. + fq(x_2 - x_1)\frac{(1-\pi)q(x_2 - x_1) - c}{\pi c^2}\right\},$$

$$\hat{I}_1 = \max\left\{x_1 - f + \frac{(1-\pi)(x_2 - x_1) - c/q}{2 - \pi}, \right.$$
$$\left. x_1 - f - \frac{\pi c}{q} + (1-\pi)(x_2 - x_1)\left[1 - fq^2\frac{(1-\pi)(x_2 - x_1 + c/q)}{\pi c^2}\right]\right\}.$$

One can verify by substitution that the three possible values are the investment levels that equate expected contracting costs for pairs of contracts from the three contracts in Proposition 1. Let $\overline{I}$ denote the upper bound in (1). Then the critical values used in Proposition 2 are $I_0 =$

min $\{\hat{I}_0, \bar{I}\}$ and $I_1 = \min\{\hat{I}_1, \bar{I}\}$. One can then verify the assertions about the optimal contract using the expected contracting costs for the three contracts of Proposition 1.

To consider mean-preserving spreads in $\tilde{x}$, let $\bar{x} = \pi x_1 + (1 - \pi)x_2$ denote the expected cash flow. Then $x_2(x_1) = (\bar{x} - \pi x_1)/(1 - \pi)$. Let $A(x_1)$, $B(x_1)$, and $C(x_1)$ denote the expected contracting costs under the single-investor truth-telling contract, two-investor truth-telling contract, and misrepresentation contract, respectively:

$$A(x_1) = f + c\frac{\pi(I + f - x_1 + c/q)}{(1 - \pi)q(x_2 - x_1 + c/q)} = f + \frac{\pi c(I + f - x_1 + c/q)}{q(\bar{x} - x_1) + (1 - \pi)c},$$

$$B(x_1) = 2f + c\frac{\pi(I + 2f - x_1)}{(1 - \pi)q(x_2 - x_1) - \pi c} = 2f + \frac{\pi c(I + 2f - x_1)}{q(\bar{x} - x_1) - \pi c},$$

$$C(x_1) = f + c\frac{\pi(I + f - x_1)}{(1 - \pi)q(x_2 - x_1) - c} = f + \frac{\pi c(I + f - x_1)}{q(\bar{x} - x_1) - c}.$$

Because $\lim_{x_1 \uparrow I+f} C(x_1) = f$ while the corresponding limits for $A(x_1)$ and $B(x_1)$ are larger than f, the misrepresentation contract is optimal when x_1 is large enough (i.e., when the distribution is safe enough). Comparing $A(x_1)$ and $C(x_1)$ and simplifying, one sees that the single-investor truth-telling contract is better than the misrepresentation contract if

$$x_1 < \left(\frac{2 - \pi}{1 - \pi}\right)(I + f) - \frac{\bar{x}}{1 - \pi}. \tag{12}$$

Assumption (2) requires $x_1 > c/q$. Under the condition of the proposition that $I > E[\tilde{x}]/(2 - \pi) + (c/q)(1 - \pi)/(2 - \pi) - f$, the right side of (12) is greater than c/q. Therefore, there are feasible values of x_1 small enough to satisfy (12). For all such values, the optimal contract exhibits truthful reporting, so truth-telling is optimal when the distribution is risky enough.

Proof of Proposition 3

I must show that when I is close enough to $x_1 - f$, the solution to Problem 1 has $\sigma_{ii} < 1$ for some i. The first step is to derive a lower bound, independent of I, on expected contracting costs under a truth-telling contract. The second step is to construct a contract that utilizes false reporting to achieve expected contracting costs below this bound when I is small.

LEMMA 1. *Any contract with truthful reporting has expected contracting costs*

$$Nf + c\sum_i \pi_i p_i > \min\left\{2f, f + \frac{\pi_1 c^2}{q(\sum_i \pi_i x_i - x_1) + c}\right\}.$$

Proof of Lemma. Let B denote the lower bound in the lemma. Any contract with multiple investors has fixed contracting costs $Nf \geq 2f$ plus some expected monitoring costs, so expected contracting costs are greater than B. Now consider any single-investor truth-telling contract, so $\sigma_{ii} = 1$ for each i and $\sigma_{ij} = 0$ for $i \neq j$. Such a contract must have $p_1 > 0$. If we had $p_1 = 0$, the constraints (6) would imply $p_i q v_{ii} + (1 - p_i q)t_i \leq t_1$ for every i. This yields a contradiction in (4), since

$$R = \sum_i \pi_i[p_i q v_{ii} + (1 - p_i q)t_i - p_i c] - f \leq \sum_i \pi_i[p_i q v_{ii} + (1 - p_i q)t_i] - f$$
$$\leq \sum_i \pi_i t_1 - f \leq \sum_i \pi_i x_1 - f = x_1 - f < I.$$

(The second-to-last inequality uses (11) and the last uses (1).) Since $p_1 > 0$, (9) implies $t_1 \leq v_{11} - c/q$. (11) requires $v_{11} \leq x_1$, so we have $t_1 \leq x_1 - c/q$.

The truthful reporting constraints (6) with $(i, j) = (i, 1)$ imply that

$$p_i q v_{ii} + (1 - p_i q)t_i \leq p_1 q v_{i1} + (1 - p_1 q)t_1 \leq p_1 q v_{i1} + (1 - p_1 q)(x_1 - c/q)$$
$$\leq p_1 q x_i + (1 - p_1 q)(x_1 - c/q),$$

where the last inequality follows from (11). Substituting this into the funding constraint (4) yields

$$I \leq R = \sum_i \pi_i[p_i q v_{ii} + (1 - p_i q)t_i - p_i c] - f \leq \sum_i \pi_i[p_i q v_{ii} + (1 - p_i q)t_i] - f$$
$$\leq \sum_i \pi_i[p_1 q x_i + (1 - p_1 q)(x_1 - c/q)] - f$$
$$= \sum_i \pi_i[x_1 - c + p_1 q(x_i - x_1 + c/q)] - f$$
$$= x_1 - c - f + p_1 q\left(\sum_i \pi_i x_i - x_1 + c/q\right).$$

Since $I > x_1 - f$ by (1), this requires $x_1 - c - f + p_1 q(\sum_i \pi_i x_i - x_1 + c/q) > x_1 - f$, or

$$p_1 > \frac{c/q}{\sum_i \pi_i x_i - x_1 + c/q}.$$

This implies expected contracting costs

$$Nf + c\sum_i \pi_i p_i \geq f + c\pi_1 p_1 > f + \frac{\pi_1 c^2}{q(\sum_i \pi_i x_i - x_1) + c} \geq B,$$

and the lemma is proved. ■

LEMMA 2. *Fix any* $\varepsilon > 0$. *For I close enough to* $x_1 - f$, *there is a contract* (*involving false reporting*) *that produces expected contracting costs* $Nf + c\sum_i \pi_i \sum_j \sigma_{ij} p_j < f + \varepsilon$.

Proof of Lemma. The proof is by construction. By assumption (2), $\sum_i \pi_i x_i > x_1 + c/q$. This implies there is a unique state k, with $2 \leq k \leq S$, that satisfies

$$\sum_1^k \pi_i x_i \geq (x_1 + c/q)\sum_1^k \pi_i \quad \text{and} \quad \sum_1^{k-1} \pi_i x_i < (x_1 + c/q)\sum_1^{k-1} \pi_i. \tag{13}$$

Let σ be defined by $\sigma = \pi_1 c / \sum_2^k \pi_i[q(x_i - x_1) - c]$, so that

$$q\sigma \sum_2^k \pi_i(x_i - x_1) = c\left(\pi_1 + \sigma \sum_2^k \pi_i\right). \tag{14}$$

The first inequality in (13) ensures that $0 < \sigma \leq 1$. The contract we will construct has the manager report truthfully when $x = x_1$ or $x > x_k$; when $x_2 \leq x \leq x_k$, the manager reports truthfully with probability $1 - \sigma$ and reports $\hat{x}_1$ with probability σ.

Fix any $p_1 > 0$, and consider a contract with transfers

$$t_1 = x_1; \text{for } i > 1, t_i = p_1 q x_i + (1 - p_1 q)x_1;$$
$$v_{ii} = t_i; \text{for } i \neq j, v_{ij} = x_i;$$

and strategies

$$\text{for } i = 1 \text{ and } i > k, \sigma_{ii} = 1;$$
$$\text{for } 2 \leq i \leq k, \sigma_{i1} = \sigma \text{ and } \sigma_{ii} = 1 - \sigma;$$
$$p_2 = \cdots = p_S = 0.$$

One can check that the manager is willing to follow the specified reporting strategies, given the transfers and monitoring probabilities. For reports $\hat{x}_j > \hat{x}_1$, the investor prefers to set $p_j = 0$ since all such reports are truthful and $v_{jj} - t_j = 0 < c/q$. Furthermore, the investor is indifferent about investigating reports $\hat{x}_1$, so is willing to set $p_1 > 0$. To check this, note from (3) that the state probabilities conditional on report $\hat{x}_1$ are

$$\gamma_{11} = \frac{\pi_1}{\pi_1 + \sigma \sum_2^k \pi_i}, \qquad \gamma_{i1} = \frac{\sigma \pi_i}{\pi_1 + \sigma \sum_2^k \pi_i} \text{ for } 2 \le i \le k,$$

and $\gamma_{i1} = 0$ for $i > k$. Therefore, the investor's expected incremental transfer from investigating report $\hat{x}_1$ is

$$\begin{aligned} q \sum_i \gamma_{i1}(v_{i1} - t_1) &= q \left[\frac{\pi_1}{\pi_1 + \sigma \sum_{i=2}^k \pi_i} (x_1 - x_1) + \sum_{i=2}^k \frac{\sigma \pi_i}{\pi_1 + \sigma \sum_{i=2}^k \pi_i} (x_i - x_1) \right] \\ &= \frac{q\sigma \sum_{i=2}^k \pi_i (x_i - x_1)}{\pi_1 + \sigma \sum_{i=2}^k \pi_i} \\ &= c \text{ using (14),} \end{aligned}$$

equal to the monitoring cost.

Given the monitoring probability p_1, this contract produces expected net revenue for the investor of

$$\begin{aligned} R(p_1) = {} & \sum_i \pi_i \sum_j \sigma_{ij} [p_j q v_{ij} + (1 - p_j q) t_j - p_j c] - f \\ = {} & \pi_1 (x_1 - p_1 c) + \sum_2^k \pi_i [\sigma \{p_1 q x_i \\ & + (1 - p_1 q) x_1 - p_1 c\} + (1 - \sigma)\{p_1 q x_i + (1 - p_1 q) x_1\}] \\ & + \sum_{k+1}^S \pi_i [p_1 q x_i + (1 - p_1 q) x_1] - f. \end{aligned}$$

Some simplification produces

$$R(p_1) = x_1 - f + p_1 \left[q \sum_1^S \pi_i (x_i - x_1) - c \left(\pi_1 + \sigma \sum_2^k \pi_i \right) \right]. \tag{15}$$

The expression in brackets is positive, as can be seen using (14) and recognizing that $\sigma \le 1$. Expected contracting costs are

$$f + c \sum_i \pi_i \sum_j \sigma_{ij} p_j = f + c \left(\pi_1 + \sigma \sum_2^k \pi_i \right) p_1. \tag{16}$$

In order to fund the project, we must have p_1 large enough that $R(p_1) \geq I$, so define p_1^* by $R(p_1^*) = I$. One can see from (15) that $p_1^* \downarrow 0$ as $I \downarrow x_1 - f$. Equation (16) then implies that expected contracting costs decrease to f as $I \downarrow x_1 - f$. Thus, for I sufficiently small, expected contracting costs are less than $f + \varepsilon$. ■

Under any truth-telling contract, expected contracting costs are greater than B, which is strictly greater than f. But as I gets close to $x_1 - f$, expected contracting costs can be made arbitrarily close to f under a contract that involves misrepresentation. Therefore, the optimal contract must involve misrepresentation when I is sufficiently small.

Proof of Proposition 4

The proposition addresses circumstances where the optimal contract involves truthful reporting and multiple investors. When truthful reporting is optimal, Problem 1 can be simplified by substituting $\sigma_{ii} = \gamma_{ii} = 1$ for all i and $\sigma_{ij} = \gamma_{ij} = 0$ for $i \neq j$, and substituting the definition of R from (4) into the objective function. To find the best *multiple-investor* contract, one can add the constraint $N \geq 2$ to the problem. This produces the following problem: Choose nonnegative numbers N, $\{t_i^n\}$, $\{v_{ij}^n\}$, $\{p_i^n\}$, and $\{p_i\}$ to solve

Problem 2.

$$\min Nf + c\sum_i \pi_i p_i$$

subject to:

$$N \geq 2$$

$$\sum_i \pi_i \left[p_i q \sum_n v_{ii}^n + (1 - p_i q) \sum_n t_i^n - p_i c \right] - Nf \geq I.$$

For each (i, j), $p_i q \sum_n v_{ii}^n + (1 - p_i q) \sum_n t_i^n \leq p_j q \sum_n v_{ij}^n + (1 - p_j q) \sum_n t_j^n$.

For each i, $\sum_n p_i^n = p_i \leq 1$.

For each i, $p_i = 1$ if there is some n with $v_{ii}^n - t_i^n > c/q$. (17)

For each i, $p_i = 0$ if $v_{ii}^n - t_i^n < c/q$ for all n. (18)

For each i, $\sum_n t_i^n \leq x_i$.

For each (i, j), $\sum_n v_{ij}^n \leq x_i$.

The task is to prove that the solution has $N = 2$ and that the nonmonitoring investor bears the cost of monitoring (through a smaller expected transfer when monitoring succeeds). The proof is accomplished by relaxing Problem 2, and then showing that the optimum of the relaxed problem can also be achieved in the more constrained setting of Problem 2. This means that a solution to Problem 2 must also solve the relaxed problem. Therefore, if some property holds for every solution to the relaxed problem, it must also hold for every solution to Problem 2.

After describing the relaxed problem, the proposition is proved using two lemmas. The first shows that any solution to the relaxed problem has $N = 2$ and has $\sum_n v_{ii}^n \leq \sum_n t_i^n$ whenever $p_i > 0$. The second shows that the optimum for the relaxed problem is attainable for Problem 2. We can therefore conclude that any solution to Problem 2 has $N = 2$ and $\sum_n v_{ii}^n \leq \sum_n t_i^n$ whenever $p_i > 0$. The second condition is used to prove that the nonmonitoring investor bears the cost of monitoring, i.e., $q(v_{ii}^2 - t_i^2) \leq -c$, where the nonmonitoring investor is labeled Investor 2.

Problem 2 is relaxed in two ways. First, we eliminate (17) from the constraints. Second, we weaken condition (18), which requires that $p_i = 0$ when $v_{ii}^n - t_i^n < c/q$ for all n. This is replaced by the requirement that $p_i = 0$ when $\sum_n v_{ii}^n < c/q$. This is a weaker condition because $\sum_n v_{ii}^n < c/q$ requires that each v_{ii}^n be less than c/q. The relaxed problem is

Problem 3.

$$\min Nf + c \sum_i \pi_i p_i$$

subject to:

$$N \geq 2$$

$$\sum_i \pi_i \left[p_i q \sum_n v_{ii}^n + (1 - p_i q) \sum_n t_i^n - p_i c \right] - Nf \geq I. \tag{19}$$

For each (i, j), $p_i q \sum_n v_{ii}^n + (1 - p_i q) \sum_n t_i^n \leq p_j q \sum_n v_{ij}^n + (1 - p_j q) \sum_n t_j^n$. (20)

For each i, $\sum_n p_i^n = p_i \leq 1$.

For each i, $p_i = 0$ if $\sum_n v_{ii}^n < c/q$. (21)

For each i, $\sum_n t_i^n \leq x_i$. (22)

For each (i, j), $\sum_n v_{ij}^n \leq x_i$. (23)

Notice that the transfers $\{t_i^n\}$ and $\{v_{ij}^n\}$ enter the relaxed problem *only* through the aggregates $\sum_n t_i^n$ and $\sum_n v_{ij}^n$. We may assume that (23) binds for $i \neq j$ since this provides maximum flexibility in (20) and has no effect on the objective or the other constraints. Thus, (20) can be restated as

$$\text{for each } (i,j), p_i q \sum_n v_{ii}^n + (1 - p_i q) \sum_n t_i^n \leq p_j q x_j + (1 - p_j q) \sum_n t_j^n. \quad (24)$$

LEMMA 3. *Any solution to Problem* 3 *has* $N = 2$ *and also has* $\sum_n v_{ii}^n \leq \sum_n t_i^n$ *when* $p_i > 0$.

Proof of Lemma. It is immediate that $N > 2$ is suboptimal. If $N > 2$, one can improve the objective by combining any two securities. For example, set $\tilde{N} = N - 1$ and define $\tilde{t}_i^{N-1} = t_i^{N-1} + t_i^N$, $\tilde{v}_{ij}^{N-1} = v_{ij}^{N-1} + v_{ij}^N$, and $\tilde{p}_i^{N-1} = p_i^{N-1} + p_i^N$. This reduces contracting costs by f, adds slack to (19), and all other constraints are unchanged.

To prove $\sum_n v_{ii}^n \leq \sum_n t_i^n$ when $p_i > 0$, suppose to the contrary that there is some state with $\sum_n v_{ii}^n > \sum_n t_i^n$ and $p_i > 0$. We will construct a new contract that improves on the existing contract, by altering the transfers and reducing the monitoring probability in this state. Stars denote the altered contract.

One can show that an efficient contract must have $\sum_n t_1^n = \sum_n v_{11}^n = x_1$ and $p_i q \sum_n v_{ii}^n + (1 - p_i q) \sum_n t_i^n \geq x_1$ for every state.[23] The first condition implies that $\sum_n t_i^n \geq x_1$, else the worst type would rather report $\hat{x}_i$ than report truthfully. Given the supposition $\sum_n v_{ii}^n > \sum_n t_i^n$ and $p_i > 0$, the second condition implies that $\sum_n v_{ii}^n > x_1 > c/q$ (the last inequality using (2)). Therefore, we have the ordering

$$x_i \geq \sum_n v_{ii}^n > \sum_n t_i^n > c/q.$$

This means that small decreases in $\sum_n v_{ii}^n$ will not violate (21) and small increases in $\sum_n t_i^n$ will not violate (22).

Choose $\sum_n t_i^{n*}$ slightly larger than $\sum_n t_i^n$ (and less than x_i) and p_i^* slightly smaller than p_i to satisfy

$$p_i^* q x_S + (1 - p_i^* q) \sum_n t_i^{n*} = p_i q x_S + (1 - p_i q) \sum_n t_i^n. \quad (25)$$

This ensures that the manager's expected payment from sending a *false* report $\hat{x}_i$ is at least as large under the altered contract as under the initial contract:

[23] These results can be proved following the logic of Lemmas 1–7 of Border and Sobel (1987).

$$p_i^* q x_j + (1 - p_i^* q) \sum_n t_i^{n*} - \left(p_i q x_j + (1 - p_i q) \sum_n t_i^n \right)$$

$$= (1 - p_i^* q) \sum_n t_i^{n*} - (1 - p_i q) \sum_n t_i^n - (p_i - p_i^*) q x_j$$

$$\geq (1 - p_i^* q) \sum_n t_i^{n*} - (1 - p_i q) \sum_n t_i^n - (p_i - p_i^*) q x_S = 0,$$

the last equality by (25). Now let $\sum_n v_{ii}^{n*}$ satisfy

$$p_i^* q \sum_n v_{ii}^{n*} + (1 - p_i^* q) \sum_n t_i^{n*} = p_i q \sum_n v_{ii}^n + (1 - p_i q) \sum_n t_i^n, \quad (26)$$

so the expected transfer following a truthful report $\hat{x}_i$ is unchanged under the altered contract. (25) implies that $\sum_n v_{ii}^{n*}$ is less than $\sum_n v_{ii}^n$. As long $\sum_n t_i^{n*}$ and p_i^* were chosen close enough to $\sum_n t_i^n$ and p_i in (25), $\sum_n v_{ii}^{n*}$ remains larger than c/q so (21) is not violated.

The altered contract satisfies all constraints and improves the objective because $p_i^* < p_i$, contradicting the supposition that $\sum_n v_{ii}^n > \sum_n t_i^n$. ■

LEMMA 4. *Let V^* denote the minimal expected contracting costs for Problem* 3. *There is a contract*, *feasible for Problem* 2, *that has expected contracting costs $Nf + \sum_i \pi_i p_i = V^*$.*

Proof of Lemma. Take any solution to Problem 3, and let the transfers and monitoring probabilities be denoted by stars. The solution to Problem 3 is then $V^* = 2f + c \sum_i \pi_i p_i^*$. We can construct a contract that is feasible for Problem 2 and has identical expected contracting costs. The contract we construct has two investors, and all monitoring is done by investor 1. For $i \neq j$, set v_{ij}^1 and v_{ij}^2 in any manner that satisfies $v_{ij}^1 + v_{ij}^2 = x_i$ to provide the manager with the maximum incentive to report truthfully. For every state i, set $p_i^1 = p_i^*$ and $p_i^2 = 0$, so the monitoring probability is $p_i = p_i^1 + p_i^2 = p_i^*$, identical to the contract that solves the relaxed problem. For states with $p_i = 0$, set $v_{ii}^1 = v_{ii}^2 = 0$, $t_i^1 = 0$ and $t_i^2 = \sum_n t_i^{n*}$. These are clearly feasible and give no investor an incentive to monitor. For states with $p_i > 0$, set $v_{ii}^1 = c/q$, $v_{ii}^2 = \sum_n v_{ii}^{n*} - c/q$, $t_i^1 = 0$ and $t_i^2 = \sum_n t_i^{n*}$. These transfers are feasible because $p_i^* > 0$ implies $\sum_n v_{ii}^{n*} \geq c/q$ by (21), so $v_{ii}^2 \geq 0$. Investor 1 is indifferent about monitoring, hence willing to monitor with probability p_i. Investor 2 has no incentive to monitor because

$$p_i^* > 0 \Rightarrow \sum_n v_{ii}^{n*} \leq \sum_n t_i^{n*} \Rightarrow v_{ii}^2 - t_i^2 = \sum_n v_{ii}^{n*} - c/q - \sum_n t_i^{n*} \leq -c/q < 0.$$

The first implication is from the previous lemma. ■

Given these two lemmas, we know that any solution to Problem 2 must have $N = 2$ (as claimed in the proposition) and must have $\sum_n v_{ii}^n \leq \sum_n t_i^n$ whenever $p_i > 0$. Consider any state with $p_i > 0$. If Investor 1 monitors, $q(v_{ii}^1 - t_i^1) \geq c$, and the implication for Investor 2 is that

$$v_{ii}^2 - t_i^2 = \sum_n v_{ii}^n - v_{ii}^1 - \left(\sum_n t_i^n - t_i^1\right)$$
$$= \left(\sum_n v_{ii}^n - \sum_n t_i^n\right) - (v_{ii}^1 - t_i^1) \leq 0 - c/q = -c/q.$$

Thus Investor 2 bears the entire cost of monitoring through a lower transfer: $q(v_{ii}^2 - t_i^2) \leq -c$. This proves the proposition.

Proofs of Propositions 5 and 6

These proofs, which are straightforward, are available from the author.

REFERENCES

Allen, F., and Gale, D. (1989). Optimal security design, *Rev. Financ. Stud.* **1,** 229–263.

Allen, F., and Winton, A. (1992). "Corporate Financial Structure, Incentives and Optimal Contracting," working paper, University of Pennsylvania and Northwestern University.

Arya, A., Woodlock, P., and Young, R. A. (1992). Managerial reporting discretion and the truthfulness of disclosures, *Econ. Lett.* **39,** 163–168.

Baiman, S., Evans III, J. H., and Noel, J. (1987). Optimal contracts with a utility-maximizing auditor, *J. Acc. Res.* **25,** 217–244.

Beck, P. J., and Jung, W. (1989). Taxpayers' reporting decisions and auditing under information asymmetry, *Accounting Rev.* **64,** 468–487.

Berglöf, E., and von Thadden, E.-L. (1994). Short-term versus long-term interests: capital structure with multiple investors, *Quart. J. Econ.* **109,** 1055–1084.

Betker, B. L. (1995). An empirical examination of prepackaged bankruptcy, *Financial Manage.* **24,** 3–18.

Bhattacharya, U., and Krishnan, M. (1995). "To Believe or Not to Believe," working paper, University of Iowa and University of Minnesota.

Bolton, P., and Scharfstein, D. S. (1996). Optimal debt structure and the number of creditors, *J. Polit. Econ.* **104,** 1–25.

Boot, A. W. A., and Thakor, A. V. (1993). Security design, *J. Finance* **48,** 1349–1378.

Border, K. C., and Sobel, J. (1987). Samurai accountant: A theory of auditing and plunder, *Rev. Econ. Stud.* **54,** 525–540.

Boyd, J. H., and Smith, B. D. (1992). Intermediation and the equilibrium allocation of investment capital: Implications for economic development, *J. Monet. Econ.* **30,** 409–432.

Boyd, J. H., and Smith, B. D. (1994). How good are standard debt contracts? Stochastic versus nonstochastic monitoring in a costly state verification environment? *J. Bus.* **67,** 539–561.

Boyd, J. H., and Smith, B. D. (1996). "The Use of Debt and Equity in Optimal Financial Contracts," working paper, Federal Reserve Bank of Minneapolis.

Chang, C. (1990). The dynamic structure of optimal debt contracts, *J. Econ. Theory* **52,** 68–86.

Dewatripont, M. (1989). Renegotiation and information revelation over time: The case of optimal labor contracts, *Quart. J. Econ.* **104,** 589–620.

Dewatripont, M., and Tirole, J. (1994). A theory of debt and equity: Diversity of securities and manager–shareholder congruence, *Quart. J. Econ.* **109,** 1027–1054.

Evans, J. H. (1980). Optimal contracts with costly conditional auditing, *J. Acc. Res.* **18,** 108–139.

Gale, D., and Hellwig, M. (1985). Incentive compatible debt contracts: The one-period problem, *Rev. Econ. Stud.* **52,** 647–663.

Gale, D., and Hellwig, M. (1989). Repudiation and renegotiation: The case of sovereign debt, *Int. Econ. Rev.* **30,** 3–32.

Gertner, R., and Scharfstein, D. (1991). A theory of workouts and the effects of reorganization law, *J. Finance* **46,** 1189–1222.

Graetz, M. J., Reinganum, J. F., and Wilde, L. L. (1986). The tax compliance game: Toward an interactive theory of law enforcement, *J. Law, Econ. and Organization* **2,** 1–32.

Green, J. R., and Laffont, J.-J. (1986). Partially verifiable information and mechanism design, *Rev. Econ. Stud.* **53,** 447–456.

Grossman, S. J., and Hart, O. D. (1986). The costs and benefits of ownership: A theory of vertical and lateral integration, *J. Polit. Econ.* **94,** 691–719.

Hart, O., and Moore, J. (1988). Incomplete contracts and renegotiation, *Econometrica* **56,** 755–786.

Hart, O., and Moore, J. (1990). Property rights and the nature of the firm, *J. Polit. Econ.* **98,** 1119–1158.

Hart, O., and Moore, J. (1995). Debt and seniority: An analysis of the role of hard claims in constraining management, *Amer. Econ. Rev.* **85,** 567–585.

Hart, O. D., and Tirole, J. (1988). Contract renegotiation and Coasian dynamics, *Rev. Econ. Stud.* **55,** 509–540.

Harris, M. H., and Raviv, A. (1989). The design of securities, *J. Finan. Econ.* **24,** 255–288.

Harris, M. H., and Raviv, A. (1991). The theory of capital structure, *J. Finance* **46,** 297–355.

Harris, M. H., and Townsend, R. M. (1981). Resource allocation under asymmetric information, *Econometrica* **49,** 33–64.

Kofman, F., and Lawarée, J. (1993). Collusion in hierarchical agency, *Econometrica* **61,** 629–656.

Krasa, S., and Villamil, A. (1992). Monitoring the monitor: An incentive structure for a financial intermediary, *J. Econ. Theory* **57,** 197–221.

Lacker, J. M. (1989). "Limited Commitment and Costly Enforcement," working paper 90-2, Federal Reserve Bank of Richmond.

Laffont, J. J., and Martimort, D. (1997). Collusion under asymmetric information, *Econometrica* **65,** 875–911.

Laffont, J. J., and Tirole, J. (1990). Adverse selection and renegotiation in procurement," *Rev. Econ. Stud.* **57,** 597–626.

Maskin, E., and Tirole, J. (1992). The principal-agent relationship with an informed principal, II: Common values, *Econometrica* **60,** 1–42.

Melumad, N. D., and Mookherjee, D. (1989). Delegation as commitment: The case of income tax audits, *Rand J. Econ.* **20,** 139–163.

Mookherjee, D., and Png, I. (1989). Optimal auditing, insurance and redistribution, *Quart. J. Econ.* **103,** 399–416.

Myerson, R. B. (1979). Incentive compatibility and the bargaining problem, *Econometrica* **47,** 61–73.

Reinganum, J. F., and Wilde, L. L. (1986). Equilibrium verification and reporting policies in a model of tax compliance, *Int. Econ. Rev.* **27,** 739–760.

Rothschild, M., and Stiglitz, J. E. (1970). Increasing risk: I. A definition, *J. Econ. Theory* **2,** 225–243.

Rubinstein, A., and Wolinsky, A. (1992). Renegotiation-proof implementation and time preferences, *Amer. Econ. Rev.* **82,** 600–614.

Tirole, J. (1986). Hierarchies and bureaucracies: On the role of collusion in organizations, *J. Law, Econ. and Organization* **2,** 181–214.

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification, *J. Econ. Theory* **21,** 265–293.

Townsend, R. M. (1988). Information constrained insurance: The revelation principle extended,'' *J. Monet. Econ.* **21,** 411–450.

Winton, A. (1992). Costly state verification and multiple investors: The role of seniority, *Rev. Finan. Stud.* **8,** 91–123.